Published by Institute of Physics Publishing for SISSA/ISAS
Received: *October 22, 2002*
Accepted: *December 2, 2002*

Brane-assisted Scherk-Schwarz supersymmetry breaking in orbifolds*

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Abstract: We have analyzed the interplay between Scherk-Schwarz supersymmetry breaking and general fermion (gaugino or gravitino) mass terms localized on the fixed point branes of the $S^1/\mathbb{Z}_2$ orbifold. Analytic solutions for eigenfunctions and eigenvalues are found in all cases. All results are checked by numerical calculations that make use of regularized δ-functions. Odd and generically also even fermions are discontinuous at the brane fixed points, but in all cases the combination that couples to the brane is continuous. For CP-even brane mass terms supersymmetry restoration can take place when their effects are cancelled by those of Scherk-Schwarz compactification. However such a cancellation can not occur for CP-odd brane mass terms.

Keywords: Field Theories in Higher Dimensions, Extra Large Dimensions, Supersymmetry Breaking.

JHEP12(2002)002

*Work supported in part by CICYT, Spain, under contracts FPA 2001-1806 and FPA 2002-00748, by EU under contracts HPRN-CT-2000-00152 and HPRN-CT-2000-00148, and by NSF under grants P420D3620414350 and P4203620434350.

Contents

1. Introduction

Extra dimensions at the TeV scale [1] have become a popular feature in nowadays model building. They can provide possible explanations to some of the present problems within the minimal supersymmetric extension of the Standard Model (MSSM) like supersymmetry breaking, the flavour structure, electroweak symmetry breaking, or even unification with gravity. In particular, one of the most appealing possibilities to break supersymmetry in theories with extra dimensions is the Scherk-Schwarz (SS) mechanism [2], based on non-trivial twists that generate a mass term through compactification [3, 4]. This mechanism appears to be natural in any locally supersymmetric five-dimensional (5D) theory, since it has been proven that the SS mechanism can be seen as a spontaneous breaking of the $\mathrm{SU}(2)_R$ via some off-shell auxiliary fields [5, 6, 7]. Being spontaneous this kind of supersymmetry breaking is quite desirable since it will not generate any quadratic divergences [8].

The most common way of compactifying extra-dimensional theories is by using orbifolds [9] which allow for chirality and partial breaking of supersymmetry; this is needed because theories in extra dimensions are in general non-chiral and have extended supersymmetry. Orbifolds introduce fixed points in the theory, where translational invariance is broken. Therefore terms localized at those points can arise without spoiling the higher dimensional Poincare invariance in the bulk. This type of localized interactions can be used to break supersymmetry dynamically, e.g. via gaugino condensation, or by any other kind of mechanism that could generate a brane mass term for gravitinos. The possibility of supersymmetry restoration between the SS mechanism and brane localized supersymmetry breakdown was already proposed in the context of the heterotic Horava-Witten M-theory [10]. Also, this kind of interactions have been studied in the past in the context of generalized SS mechanism [11], since these terms completely change the spectrum and moreover any fermion mass term localized on a brane will generate discontinuities in the wavefunctions. The aim of this paper is to generalize previous results [11, 12, 6, 13, 14] and make a systematic analysis of a very general class of models that could shed light on the mechanism of supersymmetry breaking.

In this paper we will study a 5D theory compactified on $S^1/\mathbb{Z}_2$ with SS supersymmetry breaking plus the most general fermion mass terms on the brane, which do not necessarily have to respect $\mathrm{SU}(2)_R$ since the latter is broken by the compactification. We will perform a fully analytic study of both the CP-even and the CP-odd brane mass terms and find out the cases where there are massless states in the spectrum, corresponding to situations where there is cancellation of supersymmetry breaking between the different sources. We will also show how the discontinuities of the fields are treated consistently and are independent of any regularization of the brane. We will perform all explicit calculations for the case of gauginos that are symplectic Majorana spinors transforming as doublets with respect to $\mathrm{SU}(2)_R$. Our results will be straightforwardly generalized to the case of symplectic Majorana 5D gravitinos that also transform as doublets of $\mathrm{SU}(2)_R$. All our analytic results have been numerically checked using regularized δ-functions.

The layout of the paper is as follows. In section 2 we will present the model with both the bulk lagrangian plus possible mass terms localized on the brane that will be classified according to its transformation properties under the four-dimensional CP-invariance. Section 3 will be devoted to the full analysis of the CP-even brane whereas section 4 contains the CP-odd case. Finally our conclusions will be drawn in section 5.

2. Fermion bulk lagrangian and brane-mass terms

We will consider a five-dimensional model compactified on the orbifold $S^1/\mathbb{Z}_2$ where $\mathbb{Z}_2$ transforms the fifth coordinate $x^5 \equiv y$ into $-y$. Fermions in five dimensions can be described by a pair of symplectic Majorana spinors λ^i that transform as a doublet under $\mathrm{SU}(2)_R$ and such that

$$\lambda^i = \begin{pmatrix} \lambda_L^i \\ \epsilon^{ij}\bar{\lambda}_{L\,j} \end{pmatrix} \tag{2.1}$$

where λ_L^i are the two-component spinors $\lambda_L \equiv \lambda_\alpha$ and $\bar{\lambda}_L \equiv \bar{\lambda}^{\dot{\alpha}}$. Under $\mathbb{Z}_2$ the transformation law of λ is

$$\lambda(y) \to \mathcal{P}_\lambda \sigma^3 i\gamma^5 \lambda(-y) \tag{2.2}$$

where $\mathcal{P}_\lambda = \pm$ is the intrinsic parity of the spinor λ, the matrix σ^3 is acting on $\mathrm{SU}(2)_R$ indices, $\gamma^5 = diag(-i, i)$ and we are using the metric $\eta_{MN} = diag(1,-1,-1,-1,-1)$. Then for a spinor with intrinsic parity $\mathcal{P}_\lambda = +$ it turns out that $\lambda_L^1(y)$ is an even and $\lambda_L^2(y)$ an odd two-component spinor.

We will find it convenient to define 4D Majorana spinors as

$$\psi^i = \begin{pmatrix} \lambda_L^i \\ \bar{\lambda}_{L\,i} \end{pmatrix} \tag{2.3}$$

although obviously they do not transform irreducibly under $\mathrm{SU}(2)_R$ nor are they covariant under the 5D Lorentz group. Under $\mathbb{Z}_2$ parity $\psi^1(y)$ is even and $\psi^2(y)$ odd.

The lagrangian for λ in the bulk contains

$$\mathcal{L}_{\mathrm{bulk}} = i\,\bar{\lambda}\gamma^M \mathcal{D}_M \lambda \tag{2.4}$$

where $\mathcal{D}_M$ is the covariant derivative with respect to local Lorentz and gauge transformations and summation over the $\mathrm{SU}(2)_R$ indices is always understood. The kinetic lagrangian is obtained from the 4D covariant derivative $\mathcal{D}_\mu$ as

$$\mathcal{L}_{\rm kin} = i\,\bar\lambda\gamma^\mu\partial_\mu\lambda = i\,\bar\psi\gamma^\mu\partial_\mu\psi \tag{2.5}$$

while the KK-mass bulk term comes from the ∂_5 term in $\mathcal{D}_5$ as

$$\mathcal{L}_{\rm KK-mass} = i\,\bar\lambda\gamma^5\partial_5\lambda = \bar\psi_2\partial_5\psi^1 - \bar\psi_1\partial_5\psi^2\,. \tag{2.6}$$

In the off-shell formulation of 5D supergravity [15, 16] it was proven that $\mathcal{D}_M$ also contains the covariant derivative with respect to local $\mathrm{SU}(2)_R$ tansformations, $-i\vec\sigma\vec V_M$, where $\vec V_M$ are auxiliary fields in the minimal supergravity multiplet. This fact was used in refs. [5, 6] to interpret the Scherk-Schwarz SS breaking of supersymmetry as the background configuration[1] $\langle\vec\sigma\vec V_M\rangle = \delta_{M5}\sigma^2\langle V_5^2\rangle \equiv \delta_{M5}\sigma^2\omega$. This gives rise to the SS-lagrangian

$$\mathcal{L}_{\rm SS} = i\,\omega\bar\lambda\gamma^5\sigma^2\lambda = -\omega\left(\bar\psi_1\psi^1 + \bar\psi_2\psi^2\right) \tag{2.7}$$

The bulk lagrangian can then be written as

$$\mathcal{L}_{\rm bulk} = i\,\bar\psi_i\gamma^\mu\partial_\mu\psi^i + \bar\psi_2\partial_5\psi^1 - \bar\psi_1\partial_5\psi^2 - \omega(\bar\psi_1\psi^1 + \bar\psi_2\psi^2)\,. \tag{2.8}$$

In the brane the different possible $\mathbb{Z}_2$-even mass terms can be either CP-even or CP-odd. CP-even mass terms on the brane are provided by the following contributions

$$\bar\lambda\,\sigma^2\gamma^5\lambda = -\left(\bar\psi_1\psi^1 + \bar\psi_2\psi^2\right) \tag{2.9}$$

and

$$\bar\lambda\,\sigma^1\lambda = -\left(\bar\psi_1\psi^1 - \bar\psi_2\psi^2\right) \tag{2.10}$$

or an arbitrary linear combination of (2.9) and (2.10). We will then consider possible mass terms localized at one of the branes, e.g. $y = 0$, as

$$\mathcal{L}_{\rm brane,\,CP-even} = -2\Lambda\delta(y)\left[\bar\psi_1\psi^1 + \rho\,\bar\psi_2\psi^2\right] \tag{2.11}$$

where ρ is an arbitrary real parameter.

CP-odd mass terms on the brane can be provided by the contributions

$$\bar\lambda\,\sigma^1\gamma^5\lambda = -\left(\bar\psi_1\gamma^5\psi^1 - \bar\psi_2\gamma^5\psi^2\right) \tag{2.12}$$

and

$$\bar\lambda\,\sigma^2\lambda = \bar\psi_1\gamma^5\psi^1 + \bar\psi_2\gamma^5\psi^2 \tag{2.13}$$

or any arbitrary linear combination of (2.12) and (2.13). We can then consider the brane lagrangian

$$\mathcal{L}_{\rm brane,\,CP-odd} = -2\Lambda\delta(y)\left[\bar\psi_1\gamma^5\psi^1 - \rho\,\bar\psi_2\gamma^5\psi^2\right] \tag{2.14}$$

where ρ is an arbitrary real parameter.

[1] We are using units where the compactification radius $R \equiv 1$. Restoring the R-dependence at any stage should be a trivial task.

Five-dimensional theories where brane mass terms (2.9) and (2.10), or (2.12) and (2.13), appear can be found in the recent literature. On the other hand solving theories where the brane lagrangians (2.11) and (2.14) interfere with the Scherk-Schwarz breaking has been done only for some cases. In particular the lagrangian (2.11) for $\rho = 0, 1$ [11, 13] and the lagrangian (2.14) for $\rho = 0$ [6]. In the following sections we will analyze the general cases both for (2.11) and (2.14).

3. CP-even brane mass terms

The lagrangian in this case is given by (2.8) and (2.11), i.e.

$$\mathcal{L}_{\mathrm{CP-even}} = i\,\bar{\psi}_i\gamma^\mu\partial_\mu\psi^i + \bar{\psi}_2\partial_5\psi^1 - \bar{\psi}_1\partial_5\psi^2 - \omega\left(\bar{\psi}_1\psi^1 + \bar{\psi}_2\psi^2\right) - 2\Lambda\delta(y)\left[\bar{\psi}_1\psi^1 + \rho\,\bar{\psi}_2\psi^2\right]. \tag{3.1}$$

The equations of motion (EOM) from the lagrangian (3.1) are given by

$$\begin{aligned}(-i\,\gamma^\mu\partial_\mu + \omega)\,\psi^1_n + \partial_y\psi^2_n + 2\Lambda\delta(y)\psi^1_n &= 0\\ (-i\,\gamma^\mu\partial_\mu + \omega)\,\psi^2_n - \partial_y\psi^1_n + 2\rho\,\Lambda\delta(y)\psi^2_n &= 0\,.\end{aligned} \tag{3.2}$$

We can now decompose the fields $\psi^i_n(x,y)$ in plane waves as $\psi^i_n(x,y) = \exp(-ip\cdot x)\psi^i_n(p,y)$ where $p^2 = M_n^2$, M_n being the mass eigenvalues and $\psi^i_n(p,y)$ the mass eigenfunctions. In the rest frame $p^\mu = (M_n, \vec{0})$ eq. (3.2) is written as

$$\begin{aligned}\left(-M_n\gamma^0 + \omega\right)\psi^1_n + \partial_y\psi^2_n + 2\Lambda\delta(y)\psi^1_n &= 0\\ \left(-M_n\gamma^0 + \omega\right)\psi^2_n - \partial_y\psi^1_n + 2\rho\,\Lambda\delta(y)\psi^2_n &= 0\,.\end{aligned} \tag{3.3}$$

where $\psi^i_n = \psi^i_n(M_n, y)$ are the mass eigenstates in the rest frame.

We define now Majorana spinors as

$$\psi^i_n = \begin{pmatrix} \phi^i_n(y)\ \xi_n \\ \bar{\phi}_{i\,n}(y)\ \xi_n \end{pmatrix} \tag{3.4}$$

where ξ_n are constant real spinors and $\phi^i_n(y)$ complex functions. We can then write the EOM (3.3) as

$$\begin{aligned}-M_n\bar{\phi}^1_n + \omega\phi^1_n + \partial_y\phi^2_n + 2\Lambda\delta(y)\phi^1_n &= 0\\ -M_n\bar{\phi}^2_n + \omega\phi^2_n - \partial_y\phi^1_n + 2\rho\,\Lambda\delta(y)\phi^2_n &= 0\end{aligned} \tag{3.5}$$

where the constant two-component spinors ξ_n factorize out of the EOM (3.5) and play no role in what follows.

An explicit analytic solution of equations (3.5) with real functions $\bar{\phi}^i_n(y) = \phi^i_n(y)$ can be obtained in the region $-2\pi < y < 2\pi$, for initial conditions

$$\begin{aligned}\phi^1_n(0) &= 1\\ \phi^2_n(0) &= 0\,,\end{aligned} \tag{3.6}$$

as

$$\begin{pmatrix} \phi_n^1(y) \\ \phi_n^2(y) \end{pmatrix} = \sqrt{\frac{1+t_\rho^2\left(\Lambda\varepsilon(y)\right)}{1+\rho\, t_\rho^2\left(\Lambda\varepsilon(y)\right)}} \begin{pmatrix} \cos\left[(M_n-\omega)y - \arctan\, t_\rho\left(\Lambda\varepsilon(y)\right)\right] \\ \sin\left[(M_n-\omega)y - \arctan\, t_\rho\left(\Lambda\varepsilon(y)\right)\right] \end{pmatrix} \tag{3.7}$$

where $\varepsilon(y) = \mathrm{sign}(y)$ is the sign function and we use the notation,

$$t_\rho(x) = \frac{1}{\sqrt{\rho}}\tan(\sqrt{\rho}\,x) \tag{3.8}$$

For the typical values of $\rho = 1, 0, -1$ this function is given by:

$$t_1(x) = \tan(x) \tag{3.9}$$

$$t_0(x) = x \tag{3.10}$$

$$t_{-1}(x) = \tanh(x) \tag{3.11}$$

In the region $-2\pi < y < 2\pi$ the functions (3.7) are periodic provided the condition

$$M_n = n + \omega + \frac{1}{\pi}\arctan\, t_\rho(\Lambda) \tag{3.12}$$

holds. Notice that supersymmetry can be restored if ω and Λ are such that $\omega\pi + \arctan t_\rho(\Lambda) = n\pi$, where n is any integer number in which case there is always a massless mode in the spectrum.

The solution (3.7) near the brane, i.e. for $|y| < \epsilon$ where $\epsilon \to 0$, is given by

$$\begin{pmatrix} \phi_n^1(y) \\ \phi_n^2(y) \end{pmatrix} = \frac{1}{\sqrt{1+\rho t_\rho^2\left(\Lambda\varepsilon(y)\right)}} \begin{pmatrix} 1 \\ -t_\rho\left(\Lambda\varepsilon(y)\right) \end{pmatrix} \tag{3.13}$$

From (3.13) we see that ϕ_n^2 is an odd function that makes a jump at the brane $\Delta\phi_n^2$ given by

$$\left|\Delta\phi_n^2\right| = \frac{2t_\rho(\Lambda)}{\sqrt{1+\rho\, t_\rho^2(\Lambda)}} \tag{3.14}$$

while ϕ_n^1 has a discontinuity on the brane

$$\lim_{y\to 0}\phi_n^1(y) = \frac{1}{\sqrt{1+\rho\, t_\rho^2(\Lambda)}}\,. \tag{3.15}$$

Which in general is different from $\phi_n^1(0) = 1$. Only for the particular case $\rho = 0$ is the function $\phi_n^1(y)$ continuous. However one can check that near the brane

$$\lim_{y\to 0}\left[\left(\phi_n^1(y)\right)^2 + \rho\left(\phi_n^2(y)\right)^2\right] = \left(\phi_n^1(0)\right)^2 + \rho\left(\phi_n^2(0)\right)^2 \tag{3.16}$$

and so the term in the lagrangian (3.1) coupled to the brane is continuous.

In figure 1 we present a numerical solution to eq. (3.5) obtained by regularizing the δ-function with a gaussian of width $\sigma = 10^{-3}$ (we have checked our results using other regularizations as well). The solutions precisely coincide with the analytic ones, eqs. (3.7) and (3.12). We display in detail the discontinuities found in eq. (3.14) and eq. (3.15). Furthermore we plot in figure 2 the combination $(\phi_n^1(y))^2 + \rho\,(\phi_n^2(y))^2$. Note that the scales used in figure 2 and in figure 1 (right panel) are identical and the continuity of $(\phi_n^1(y))^2 + \rho\,(\phi_n^2(y))^2$ is explicit.

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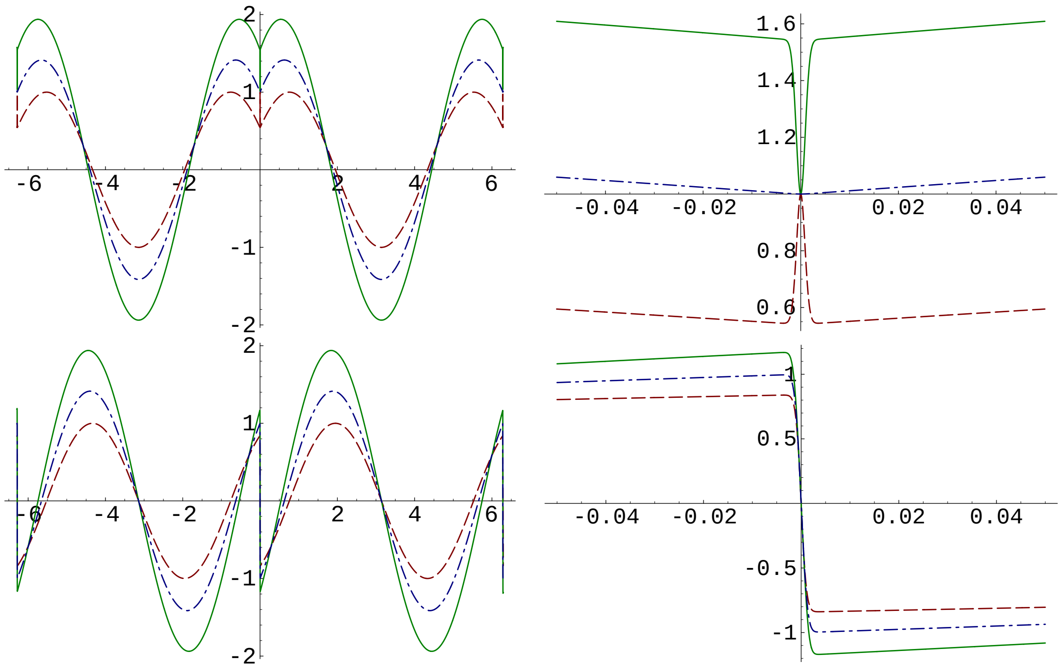

Figure 1: The numerical solution of eq. (3.5) for $\Lambda = 1$ and $\omega = 0.2$. We show three different values of ρ, $\rho = -1$ (solid line), $\rho = 1$ (dashed line) and $\rho = 0$ (dashed-dotted line). In the right plot we display in detail the different discontinuities.

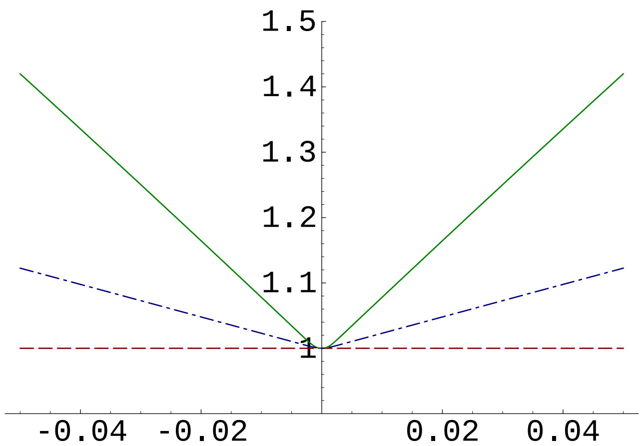

Figure 2: The quantity $(\phi_n^1(y))^2 + \rho\,(\phi_n^2(y))^2$ taken from the numerical solution with a gaussian-regularized δ-function near the origin. Again, we diplay the cases $\rho = -1$ (solid line), $\rho = 1$ (dashed line) and $\rho = 0$ (dashed-dotted line). In all cases the quantity is continous in accordance with the analytical solution.

If we replace $\delta(y) \to \sum_k \delta(y - 2\pi k)$ in (3.1) the solution (3.7) can be generalized to one periodic for $y \in \mathbb{R}$ as

$$\begin{pmatrix} \phi_n^1(y) \\ \phi_n^2(y) \end{pmatrix} = \sqrt{\frac{1 + t_\rho^2\,(\Lambda\beta(y))}{1 + \rho\, t_\rho^2\,(\Lambda\beta(y))}} \begin{pmatrix} \cos\left[(M_n - \omega)y - \widetilde{\eta}(y)\right] \\ \sin\left[(M_n - \omega)y - \widetilde{\eta}(y)\right] \end{pmatrix} \tag{3.17}$$

where

$$\beta(y)=\lim_{\kappa\to\infty}\sum_{k=-\kappa}^{\kappa}(-)^k\varepsilon(y-2\pi k)=\begin{cases}+1 & 2n<\frac{y}{2\pi}<(2n+1)\\ 0 & \frac{y}{2\pi}=n\\ -1 & 2n+1<\frac{y}{2\pi}<(2n+2)\end{cases}\tag{3.18}$$

is an odd function that makes the prefactor periodic along the whole real axis. The function $\widetilde{\eta}(y)$ is given by

$$\widetilde{\eta}(y)=\lim_{\kappa\to\infty}\sum_{k=-\kappa}^{\kappa}\arctan\ t_\rho\left(\Lambda\varepsilon(y-2\pi k)\right).\tag{3.19}$$

Notice that the function (3.19) has the same shape as

$$\eta(y)\arctan t_\rho(\Lambda)$$

where $\eta(y)$ is the staircase function

$$\eta(y)=\lim_{\kappa\to\infty}\sum_{k=-\kappa}^{\kappa}\varepsilon(y-2\pi k)=\begin{cases}2n & \frac{y}{2\pi}=n\\ 2n+1 & n<\frac{y}{2\pi}<n+1\end{cases}\tag{3.20}$$

that steps by two units every 2π. However obviously both functions differ by their derivatives on the brane (in the special case $\rho=1$ the two functions are indeed identical, $\widetilde{\eta}(y)=\Lambda\eta(y)$).

It is a simple matter to prove that the functions (3.17) are periodic in the real axis $\mathbb{R}$ provided that condition (3.12) holds.

4. CP-odd brane mass terms

The lagrangian in this case is given by (2.8) and (2.14), i.e.

$$\mathcal{L}_{\mathrm{CP-odd}}=i\,\bar{\psi}_i\gamma^\mu\partial_\mu\psi^i+\bar{\psi}_2\partial_5\psi^1-\bar{\psi}_1\partial_5\psi^2-\omega\left(\bar{\psi}_1\psi^1+\bar{\psi}_2\psi^2\right)-2\Lambda\delta(y)\left[\bar{\psi}_1\gamma^5\psi^1-\rho\,\bar{\psi}_2\gamma^5\psi^2\right].\tag{4.1}$$

We can then write the EOM in the rest frame $p^\mu=(M_n,\vec{0})$ as

$$\begin{aligned}\left(-M_n\gamma^0+\omega\right)\psi_n^1+\partial_y\psi_n^2+2\Lambda\delta(y)\gamma^5\psi_n^1&=0\\ \left(-M_n\gamma^0+\omega\right)\psi_n^2-\partial_y\psi_n^1-2\rho\,\Lambda\delta(y)\gamma^5\psi_n^2&=0\,.\end{aligned}\tag{4.2}$$

where ψ_n^i are the mass eigenstates corresponding to the mass eigenvalues M_n. If we define now the Majorana spinors as in (3.4) we can write the EOM (4.2) as

$$\begin{aligned}-M_n\bar{\phi}_n^1+\omega\phi_n^1+\partial_y\phi_n^2+2i\Lambda\delta(y)\phi_n^1&=0\\ -M_n\bar{\phi}_n^2+\omega\phi_n^2-\partial_y\phi_n^1-2i\rho\,\Lambda\delta(y)\phi_n^2&=0\,.\end{aligned}\tag{4.3}$$

Unlike eqs. (3.5), eqs. (4.3) do not admit real solutions. In fact by decomposing

$$\phi_n^i=\varphi_n^i+i\,\chi_n^i\tag{4.4}$$

eqs. (4.3) lead to

$$\begin{aligned} -(M_n-\omega)\varphi_n^1+\partial_y\,\varphi_n^2+2\Lambda\delta(y)\chi_n^1 &= 0\\ (M_n+\omega)\chi_n^1+\partial_y\,\chi_n^2-2\Lambda\delta(y)\varphi_n^1 &= 0\\ -(M_n-\omega)\varphi_n^2-\partial_y\,\varphi_n^1-2\rho\,\Lambda\delta(y)\chi_n^2 &= 0\\ (M_n+\omega)\chi_n^2-\partial_y\,\chi_n^1+2\rho\,\Lambda\delta(y)\varphi_n^2 &= 0\,. \end{aligned} \tag{4.5}$$

Eqs. (4.5) can be solved analytically. We choose the most general initial conditions consistent with parity (and up to a global normalization factor):

$$\begin{aligned} \varphi_n^1(0) &= \zeta\,,\\ \chi_n^1(0) &= 1\,,\\ \varphi_n^2(0) &= 0\,,\\ \chi_n^2(0) &= 0\,. \end{aligned} \tag{4.6}$$

In the interval $-2\pi<y<2\pi$ an analytic solution is then given by

$$\begin{pmatrix}\varphi_n^1(y)\\ \varphi_n^2(y)\end{pmatrix}=\sqrt{\frac{\zeta^2+t_\rho^2\left(\Lambda\varepsilon(y)\right)}{1+\rho\;t_\rho^2\left(\Lambda\varepsilon(y)\right)}}\begin{pmatrix}\cos\left[(M_n-\omega)y-\arctan\left\{t_\rho\left(\Lambda\varepsilon(y)\right)/\zeta\right\}\right]\\ \sin\left[(M_n-\omega)y-\arctan\left\{t_\rho\left(\Lambda\varepsilon(y)\right)/\zeta\right\}\right]\end{pmatrix} \tag{4.7}$$

$$\begin{pmatrix}\chi_n^1(y)\\ \chi_n^2(y)\end{pmatrix}=\sqrt{\frac{1+\zeta^2\,t_\rho^2\left(\Lambda\varepsilon(y)\right)}{1+\rho\,t_\rho^2\left(\Lambda\varepsilon(y)\right)}}\begin{pmatrix}\cos\left[(M_n+\omega)y-\arctan\left\{\zeta\;t_\rho\left(\Lambda\varepsilon(y)\right)\right\}\right]\\ -\sin\left[(M_n+\omega)y-\arctan\left\{\zeta\;t_\rho\left(\Lambda\varepsilon(y)\right)\right\}\right]\end{pmatrix} \tag{4.8}$$

where both ζ and M_n are to be determined by imposing periodicity of the solution.

Periodicity condition of (4.7) and (4.8) is implemented by

$$M_n=n+\frac{1}{\pi}\arctan\sqrt{\frac{\tan^2(\omega\pi)+t_\rho^2(\Lambda)}{1+\tan^2(\omega\pi)\,t_\rho^2(\Lambda)}} \tag{4.9}$$

and the initial condition is fixed by

$$\zeta=\frac{\tan(M_n+\omega)\pi}{t_\rho(\Lambda)}=\frac{t_\rho(\Lambda)}{\tan(M_n-\omega)\pi} \tag{4.10}$$

where the last equality is implied by (4.9) and ζ turns out to be independent of n. Notice that, unlike the case with CP-even brane masses, when CP-odd brane masses are present supersymmetry restoration is not possible for non-trivial values of the parameters ω and Λ.

The solutions (4.7) and (4.8) near the brane, $|y|<\epsilon$ where $\epsilon\to 0$, are given by,

$$\begin{pmatrix}\varphi_n^1(y)\\ \varphi_n^2(y)\end{pmatrix}=\frac{1}{\sqrt{1+\rho\;t_\rho^2\left(\Lambda\varepsilon(y)\right)}}\begin{pmatrix}\zeta\\ -t_\rho\left(\Lambda\varepsilon(y)\right)\end{pmatrix} \tag{4.11}$$

$$\begin{pmatrix}\chi_n^1(y)\\ \chi_n^2(y)\end{pmatrix}=\frac{1}{\sqrt{1+\rho\;t_\rho^2\left(\Lambda\varepsilon(y)\right)}}\begin{pmatrix}1\\ \zeta\,t_\rho\left(\Lambda\varepsilon(y)\right)\end{pmatrix}. \tag{4.12}$$

From (4.11) and (4.12) we see that φ_n^2 and χ_n^2 are odd functions that make jumps at the brane $\Delta\varphi_n^2$, $\Delta\chi_n^2$ given by

$$\left|\Delta\varphi_n^2\right| = \frac{2t_\rho(\Lambda)}{\sqrt{1+\rho\, t_\rho^2\,(\Lambda)}} \tag{4.13}$$

$$\left|\Delta\chi_n^2\right| = \frac{2\zeta\, t_\rho(\Lambda)}{\sqrt{1+\rho\, t_\rho^2\,(\Lambda)}} \tag{4.14}$$

while φ_n^1 and χ_n^1 have discontinuities on the brane

$$\lim_{y\to 0}\varphi_n^1(y) = \frac{\zeta}{\sqrt{1+\rho\, t_\rho^2(\Lambda)}} \tag{4.15}$$

$$\lim_{y\to 0}\chi_n^1(y) = \frac{1}{\sqrt{1+\rho\, t_\rho^2(\Lambda)}}\,. \tag{4.16}$$

Only for the particular case $\rho = 0$ are the functions $\varphi_n^1(y)$ and $\chi_n^1(y)$ continuous. However one can check that near the brane

$$\lim_{y\to 0}\left[\varphi_n^1(y)\,\chi_n^1(y) - \rho\,\varphi_n^2(y)\,\chi_n^2(y)\right] = \varphi_n^1(0)\,\chi_n^1(0) - \rho\,\varphi_n^2(0)\,\chi_n^2(0) \tag{4.17}$$

and so the term coupled to the brane in the lagrangian (4.1) is continuous.

If we replace $\delta(y) \to \sum_k \delta(y-2\pi k)$ in (4.1) the solution (4.7) and (4.8) can be generalized to one periodic for $y\in\mathbb{R}$ by replacing $\varepsilon(y)\to\beta(y)$ in the prefactors of (4.7) and (4.8) and by replacing

$$\begin{aligned}
\arctan\,\zeta\, t_\rho[\Lambda\varepsilon(y)] &\to \lim\nolimits_{\kappa\to\infty}\textstyle\sum_{k=-\kappa}^{\kappa}\arctan\,\zeta\, t_\rho[\Lambda\varepsilon(y-2\pi k)]\\
\arctan\,\frac{1}{\zeta}\, t_\rho[\Lambda\varepsilon(y)] &\to \lim\nolimits_{\kappa\to\infty}\textstyle\sum_{k=-\kappa}^{\kappa}\arctan\,\frac{1}{\zeta}\, t_\rho[\Lambda\varepsilon(y-2\pi k)]
\end{aligned} \tag{4.18}$$

in the corresponding arguments of the trigonometric functions. Using (4.9) and (4.10) one can check that the new functions are periodic along the real axis.

As in the case of CP-even mass terms we have checked our solution numerically with a regularized δ-function. We find perfect agreement with our analytical solution, eqs. (4.7)–(4.10).

5. Conclusions

In this paper we have studied the compactification of a 5D theory on $S^1/\mathbb{Z}_2$ with SS supersymmetry breaking and the most general brane mass term. We have focussed on the case of bulk symplectic Majorana fermions and so our results apply to the case of both gauginos and gravitinos. In all cases fermions obey first order differential equations and brane terms correspond to discontinuities on the fields.

We have studied in full detail both the CP-even and CP-odd brane mass cases. It has been shown that in the former case there can be a cancellation between the contribution of the standard SS and the brane terms and so supersymmetry can be restored whereas in the latter case this is not possible. Although in most cases there is a discontinuous behaviour for the even fermions near the brane, we have checked that the combination that couples to the brane is continuous throughout the orbifold fixed points. The generalization of this mechanism to a more general framework with brane fields, or to warped scenarios is left for future investigation.

Acknowledgments

We would like to thank J. Bagger and M. Redi for discussions. One of us (A.D.) wants to thank the Aspen Center for Physics where part of this work was done. G.G. thanks the DAAD for financial support.

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